

INDIAN MARITIME UNIVERSITY
(A Central University, Government of India)

May/ June 2017 End Semester Examinations
B.Tech. (Marine Engineering) Second Semester
(AY 2009-2014 batches)

Mathematics - II (UG11T1202/ UG11T2202)

Date : 13.06.2017

Maximum Marks: 100

Time: 3 Hrs

Pass Marks : 50

Note: Use of approved type scientific calculator is permitted.

PART – A (10 x 3=30 Marks)

(All questions are compulsory)

1.(a) Find a_0 for the Fourier series expansion of the function

$$f(x) = x^2 - 2 ; \quad -2 < x < 2$$

(b) Find the Laplace Transform of $e^{-3t} \sin 5t \sin 3t$.

(c) Find the Inverse Laplace Transform of $\frac{s}{(2s-1)(3s-1)}$.

(d) Solve the differential equation: $2y' \cos x + 4y \sin x = \sin 2x$.

(e) Find the particular integral of the differential equation:

$$\frac{d^2x}{dt^2} + 2 \frac{dx}{dt} + 3x = \sin t .$$

(f) Find the orthogonal trajectory of the family of semi-cubical

parabolas $ay^2 = x^3$.

(g) If 3 of 20 tyres in storage are defective and 4 of them are randomly chosen for inspection (that is, each tyre has the same chance of being selected) what is the probability that only one of the defective tyres will be included?

- (h) In a game, a player tosses 3 fair coins. He wins Rs.10 if 3 heads occur, Rs. 5 if 2 heads occur, Rs.2 if only 1 head occur and losses Rs.15 if no heads occur. What is his expected gain?
- (i) Point out the fallacy of the statement. The Mean of Binomial distribution is 3 and variance 5.
- (j) Find mode and median of given probability distribution.

X :	1	2	3	4	5
P(X):	0.2	0.3	0.1	0.1	0.3

PART – B (5 x 14=70 marks)
(Answer any FIVE of the following)

2. (a) Obtain the Fourier series for the function $f(x) = x^2$; $-\pi \leq x \leq \pi$, Sketch the graph of $f(x)$. Hence show that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$
(b) Obtain half range sine series for $f(x) = e^x$ in $0 < x < 1$. (8+6 marks)
3. (a) Find the Laplace Transform of $\frac{1-e^t}{t}$.
(b) Evaluate $\int_0^\infty \frac{\cos 6t - \cos 4t}{t} dt$, using Laplace Transform.
(c) Find the Inverse Laplace Transform of $\frac{s+3}{s^2-4s+13}$.
(d) Find the Inverse Laplace Transform using Convolution Theorem of $\frac{1}{s^2(s^2+a^2)}$. (3.5 X 4 marks)
4. (a) Solve the following differential equation :
(i) $x^4 \frac{dy}{dx} + x^3 y + \operatorname{cosec}(xy) = 0$.
(ii) $(x^2 y - 2xy^2)dx - (x^3 - 3x^2 y)dy = 0$
(b) $x^2 \frac{d^2 y}{dx^2} + 4x \frac{dy}{dx} + 2y = \log x$. (3.5 x x 4 marks)
5. (a) Solve : $\frac{d^2 y}{dx^2} + 2y = x^2 e^{3x} + e^x \cos 2x$
(b) A horizontal tie-rod of length $2l$ with concentrated load W at the centre and ends freely hinged, satisfies the differential equation $EI \frac{d^2 y}{dx^2} = Py - \frac{W}{2} x$. With conditions $x = 0, y = 0$ and $x = l, \frac{dy}{dx} = 0$, prove that the deflection δ and the bending moment M at the centre ($x = l$) are given by $\delta = \frac{W}{2Pn} (nl - \tanh nl)$ and $M = -\frac{W}{2n} \tanh nl$, where $n^2 EI = P$. (7+7 marks)
6. (a) A company has two plants to manufacture scooters. Plant I manufactures 80% of the scooters and Plant II manufactures 20% .

At plant I, 85 out of 100 scooters are rated standard quality or better. At plant II, only 65 out of 100 scooters are rated standard quality or better.

- (i) What is the probability that scooter selected at random came from plant I if it is known that the scooter is of standard quality?
- (ii) What is the probability that scooter selected at random came from plant II if it is known that the scooter is of standard quality?

(b) A random variable x has a following probability distribution

X	1	2	3	4	5	6	7
P(x)	k	2k	3k	k^2	$k^2 + k$	$2k^2$	$4k^2$

- (i) Find k (ii) $P(x > 5)$ (iii) $P(x = 1/2 < x < 5/2)$ (6+8 marks)

7. (a) A random variable X has the Probability density function

$$f(x) = k(1+x) \quad ; \quad 2 \leq x \leq 5$$

$$= 0 \quad ; \quad \text{otherwise}$$

Find (i) k (ii) $P(x > 4)$ (iii) mean (iv) variance of x

- (b) Experience has shown that 30% of rocket launching at NASA base have to be delayed due to weather conditions. Find the probabilities that among 10 rocket launching at that base :- (i) at most 3 (ii) at least 6, will have to be delayed due to weather conditions. (8+6 marks)

8.(a) In a certain factory producing cycle tyres there is a small chance 1 in 500 for any tyre to be defective. The tyres are supplied in the lots of 20. Using Poisson's distribution calculate the approximate number of lots containing no defective, one defective and two defective tyres respectively in a consignment of 20,000 tyres.

- (b) The sizes of 10,000 items are Normally distributed with mean 20 cms and S.D. 4 cms. Find the probability that an item selected at random will have a size between (i) 18 cms and 23 cms (ii) above 26 cms. (8+6 marks)

Note : Q.8(b) Where Area under standard normal probability curve is :

- (i) $P(0 < z < 0.5) = 0.1915$
 (ii) $P(0 < z < 0.75) = 0.2734$
 (iii) $P(0 < z < 1.5) = 0.4332$
